

DESIGN AND DEVELOPMENT IMPLICATIONS OF HIGH INERTIA EARTH-SCIENCE MECHANISM ACTUATOR

Scott STARIN⁽¹⁾, Evan HARRINGTON⁽¹⁾,

⁽¹⁾Avior Control Technologies, Inc, Longmont CO, USA: ssstarin@aviorcontrols.com,
eharrington@aviorcontrols.com

1. ABSTRACT

Stepper motors have been used extensively in spacecraft mechanism and solar array drive applications. High Inertia Filter Wheel applications can be challenging with relatively high moment of inertia and high-velocity filter position response requirements. This paper will discuss the design implications and development efforts for a mechanism drive actuator used in geosynchronous weather and Earth-science instrumentation.

2. KEY SYSTEM REQUIREMENTS

The customer required a common actuator for multiple proprietary applications. The actual numbers have been modified from the actual application, but the systematic process of analysis is being presented. Several key system requirements for this application are:

- (J_L) Load Inertia: 0.5 kgm²
- (J_F) Inertia Factor: ≤ 3.0
- (F_L) Load Friction: 56 mNm, nominal
- (ω_L) Velocity Under Load: ≥ 0.50 RPM.
- Positive Torque Margin per GSFC-STD-7000
- ($\Delta\theta_L$) Step Resolution: ≤ 5 arc-minutes
- Output Position Knowledge Required
- Output Position and Sensor Repeatability: ≤ 3 Arc-Minutes
- Motor Winding and Output Position Sensor Electrical Redundancy
- Actuator Height from Mounting Surface: ≤ 65 mm
- Operational Temperature -40° C to +65° C
- Actuator Mass: ≤ 1.0 kg
- Actuator Life: 1.25×10^8 steps
- Reliability of ≥ 0.997 for 1.25×10^8 steps

3. ANALYSIS OF TRANSMITTED TORQUES

The challenging overall system requirements led to several innovative and unique solutions for the application. Perhaps one of the most demanding of these is the reliability and number of steps requirement. Even though the load friction is minimal, there is considerable transmitted torque through the gearbox to accelerate the load inertia. We will assess the methodologies used in determination of transmitted torques, as well as several other key requirements.

3.1 Calculation of Torque Margins

The methodology of calculating Torque Margin per GSFC-STD-7000 is discussed in detail in Ref. [1], and not duplicated here. The Torque Margin methodology presented in Ref. [1] results in a Margin of Safety (MoS) of 0.136 at CDR and 0.372 at ATP. These numbers may sound low, compared to other margin techniques, but the NASA standard provides significant margin to the individual components of torque and requires a resultant positive (greater than zero) MoS. An implication of the NASA standard is that hardware is designed at CDR, but the performance required at ATP is lower than what the hardware was designed to meet. While it is tempting to think that more torque is always better, there are other implications of excessive torque margin to consider. Excessive torque margin will result in higher power consumption and increased pulse transients through the mechanical system. Ideally, systems should implement current limiting so that peak torques at the time of system integration can be set for actual measured levels. Setting current limits at the integration phase also provides the benefit of verifying magnitudes of torques so the lowest power consumption can be realized while maintaining acceptable torque margins.

3.2 Calculation of Load Acceleration Torques

One of the often-overlooked implications of high torque margin in driving inertial loads is the acceleration torques at each step of the stepper motor. These torque pulses can be significant and must be considered in the mechanical structural and life analysis. For this application, the load inertia is coupled directly to the output shaft, supported by a separate bearing system. The load friction is extremely low, as defined in Key System Requirements. The driven load inertia, however, is relatively high at 0.50 kgm². Each step of the stepper motor will accelerate the load, and this will translate a reaction torque throughout the actuator. The motor current is limited to maintain a maximum holding output torque of 35 Nm. This translates to a torque at low pulse rate ($T_{PPS,0}$) of 43 mNm at the motor. Using the torque at low pulse rate for these calculations will provide the *mean acceleration* and torque during each step. The Peak Torque and acceleration will be 41% higher than the mean calculated values.

For applications that do not implement current limiting, nominal and maximum motor torque calculations should

be analyzed. That is, nominal voltage, resistance and temperature for the nominal case, and minimum temperature and resistance as well as maximum torque constant and voltage for the maximum case. The engineer needs to make sure that they have structural and endurance margin at maximum conditions, but it is also of value to assess the nominal conditions to gage how conservative the analysis is. Since torque margin requirements are calculated at the minimum conditional values, the maximum conditions may result in surprising results.

The mean acceleration at the load (α_L) is calculated in Eq. 1. The mathematical proof of this equation is available by contacting the authors.

$$\alpha_L = \frac{(T_{PPS-0} - F_M)(N \cdot \eta_G) - F_L}{J_L + (J_M \cdot N^2 \cdot \eta_G)} \dots\dots\dots(1)$$

Where: (Application Values)

- α_L = Mean Acceleration of the Load (To be calculated)
- T_{PPS-0} = Torque at Low Pulse Rate at Motor (43 mNm)
- F_M = Detent Plus Friction at the Motor (8.5 mNm)
- N = Gear Ratio (576:1)
- F_L = Load Friction (56 mNm)
- J_L = Load Inertia (0.50 kgm²)
- J_M = Motor Rotor Inertia (8.6E-07 kgm²)
- η_G = Gearbox Efficiency (88%)

When applying these calculations, the acceleration at the load at each pulse is 23.1 Rad/sec². The Torque to Accelerate the Load ($T_{\alpha L}$) is calculated by Eq. 2:

$$T_{\alpha L} = J_L \cdot \alpha_L \dots\dots\dots(2)$$

When calculating the Mean Torque to Accelerate the load, we achieve a surprising 11.6 Nm. This torque is actually transmitted through the gearbox at every pulse of the system. The Peak torque at each pulse is 16.3 Nm. High bandwidth torque transducers have empirically verified these numbers. As a note, since the gearbox efficiency attenuates the acceleration at the output, it is most conservative to use 100% gearbox efficiency. In this subject application, we actually verified total dynamic gearbox efficiency of 88% of the four-stage gearbox. Additionally, the verified acceleration torque pulses at the load matched our analysis, verifying the dynamic gearbox efficiency via multiple methodologies.

3.3 Calculation of Loaded Time and Mean Loaded Velocity

Now that we have determined the peak-transmitted

torque to the load, we must determine the *time* the unit is under load. Stepper motors do not transmit torque between step pulses when the shaft is settled. As long as the inertia factor is reasonable (under 3.0) the overshoot and stabilization torques are also insignificant. Therefore, we are primarily interested in the *time* it takes to accelerate the load at each pulse ($t_{\alpha L}$). Which is estimated in Eq. 3.

$$t_{\alpha L} = \sqrt{\frac{2 \cdot \Delta\Theta_L}{\alpha_L}} + \tau_e \dots\dots\dots(3)$$

Where: (Application Values)

- $t_{\alpha L}$ = Time to Accelerate Load (To be calculated)
- $\Delta\Theta_L$ = Step Size at Load (9.1E-4 Radians)
- α_L = Acceleration at Load (23.1 Rad/sec²)
- τ_e = Motor Electrical Time Constant (3.0E-04 sec)

This results in a time to accelerate the load at each pulse of $t_{\alpha L} = 9.16 \text{ E-}03$ seconds. Given the life requirement of $1.25\text{E}+08$ steps, this translates in a Loaded-Lifetime requirement of about $\Sigma t_{\alpha L} = 318$ hours at 11.6 Nm. There are several offsetting secondary and tertiary components that affect the actual load characteristics and step kinematics, however, this analysis is considered conservative and appropriate. System drive electronics, internal damping characteristics as well as backlash will have minor affects on the step kinematics. These system variables can be difficult to predict or quantify, and their affects are much less significant compared to the primary variables defined in Eqs. 1-3.

Now that the torque and time components are established, we must determine the mean velocity when the mechanical energy is transferred. This calculation is presented in Eq. 4.

$$\omega_{L_Loaded} = \frac{\Delta\Theta_L}{t_{\alpha L}} \dots\dots\dots(4)$$

Where: (Application Values)

- ω_{L_Loaded} = Mean Loaded Velocity (To be calculated)

This analysis results in a Mean Loaded Velocity of $\omega_{L_Loaded} = 0.099$ Radians per second, or 0.104 RPM. The analyzed load acceleration torques ($T_{\alpha L}$), Mean Loaded Velocities, and Loaded Lifetime that must be used in the gearbox and bearing endurance and analysis. The most important factor that we are communicating here is that *the torque and velocity that is transmitted through stepper motor actuators are typically significantly higher than the load friction and step-rate velocity requirements. Increasing torque motorization*

margin will necessarily increase the torque transmitted through the gearbox, regardless of the magnitude of the load frictional components.

Another key take-away is that the Loaded Lifetime can be orders of magnitude less than the operational time, since stepper motor actuators are not transmitting torque during idle and settled times between steps. Additionally, if you run through the numbers, you will see that lower gear ratios will reduce T_{oL} at the actuator output. Doing this will increase your inertia factor and increase the probability of unstable step performance, as described in Ref. [1].

4. DEVELOPMENT OF INTEGRAL POSITION SENSOR

The application requirements include integral output position feedback as well as limitations of the height and mass of the actuator. Integrating repeatable and reliable position feedback is a highly desired option for mechanism applications. Packaging limitations and sensor options can often be driving requirements in configurations. Some heritage applications utilize anti-backlash gearing to separate components. This additional gearing and complexity can significantly decrease reliability while increasing cost and mass. Avior has addressed these concerns with an innovative line of *OnAxis* position transducers that integrate directly to the output shaft of the low speed gearbox. The customer preferred to use Brushless Resolvers for this application, although other sensor types are available. The maximum height requirement of 65 mm necessitated a right angle gearbox for this application. The integral actuator is shown in Fig. 1.



Figure 1

Actuator with Integral OnAxis Output Position Sensor

With the incorporation of the *OnAxis* position sensor, the subject actuator eliminated many failure modes associated with anti-backlash designs. Further, the variable reluctance configuration also eliminated the need for a rotary transformer, reducing the number of windings as well as the axial length. A disadvantage of the variable reluctance configuration of the resolver is

that the *accuracy* of the resolver itself is only on the order of two to three degrees, however, the *repeatability* was verified to be within the measured backlash of the gearbox of less than 3 arc-minutes. Since the system had digital processing capabilities, a simple look-up-table provided the required absolute position knowledge of the system. The configuration of the actuator met the 65 mm height and 1 kg mass requirements.

5. RELIABILITY ANALYSIS

There are many statistical approaches to calculate failure rates of components and systems. Many of which do not adequately assess the failure rates of electro-mechanical systems such as the actuator in this application. It is also impractical to manufacture a statistically significant number of units to determine failure rates and probabilities. The authors believe the approach presented herein is much more appropriate and statistically valid for mechanisms and components manufactured for the space industry.

5.1 Reliability Block Diagram

Fig. 2 breaks down the components of the actuator in a Reliability Block Diagram. The motor may be driven through either the primary or redundant motor winding. The single-string gearbox components are typical for the industry, as differential gearing adds significant cost, mass and complexity. The output position may be measured by either the primary or redundant sensor windings. Reliability Block Diagrams may seem trivial, but it provides a graphic representation of the system configuration and aids in the determination of potential failure modes and their affects. Also of note for this application is the fact that the mechanism is still functional, even with the loss of both resolvers. Step counting or optical performance data may be used to determine load position.

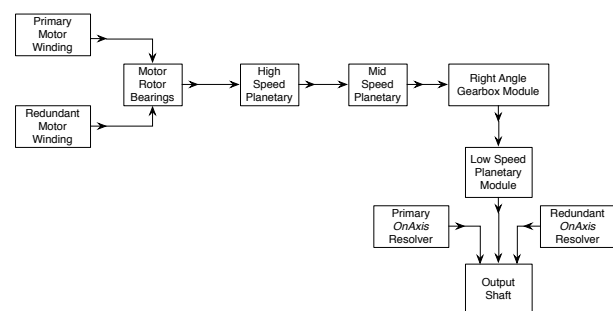


Figure 2 –
Reliability Block Diagram

5.2 Reliability Analysis of Each Component:

The methodologies of the reliability of the components are presented in the 5.3.1 of this paper. The detail summaries are shown in Appendix A herein. This methodology uses a combination of estimated L_2 life analysis, combined with classical Weibull reliability

statistical analysis. We must first determine the L_2 of the Mechanical and Electrical components of the actuator. Note: Avior uses L_2 for most space flight hardware analysis, although mathematically, you will achieve the similar reliability figures for Characteristic Life and MTBF if you use L_{10} values.

5.2.1 Gears and Bearings:

Structural bending fatigue analysis is conducted on the gears, per AGMA 2001. The analysis is conducted to determine how many cycles and hours of operation at the mean torque and velocities described in section 3 herein may be achieved. We calculate the L_2 life each of the mechanical gearing elements in the gearbox, using the AGMA equations. This calculated value tabulated in the Appendix A table. The authors conservatively use one million hours of L_2 life for each module if the calculated value is greater than one million hours.

Bearings are similarly calculated for a factored L_2 life using classical tools and software. In the right angle gearbox, the bearings see thrust, tangential and radial loads, due to the bevel gear mesh. If there are pure-torque modules in the actuator, conservative estimates of radial loads are provided, due to imperfect gear meshing and load sharing in the planetary gearboxes. Typically, 10 to 50 Newtons of radial force at the extreme of the gear mesh is assumed for the supporting bearing loads. Planet gear bearings see radial loads through the transmitted torque at each stage of gearing.

5.2.3 Brancato Method of Motor Winding Life:

Winding life estimates are analyzed using methods described in the Brancato Method, Ref. [3]. Avior’s Class H220 Insulation system is rated at 20,000 hours minimum regression life at +220° C. The **percentage of Life** at a conservatively estimated Hot Spot Temperature of +120° C is as follows:

$$L_w = 100 \cdot 2^{\frac{T_R - T_{HS_M}}{10}} \dots\dots\dots(5)$$

Where,

- L_w = Percentage of Life (To Be Calculated)
- T_R = Rated Insulation System (+220° C)
- T_{HS_M} = Motor Winding Hot-Spot Temperature (+120° C)

This equates to a lifetime of **102,400 percent greater** than the minimum regression rating of 20,000 hours. This translates to 2.05 E+07 Hours of life for the winding insulation system. In this application, Avior used an attenuation factor of 0.1 or 2.05 E+06 hours, for motor and sensor windings used in Geosynchronous and Deep-Space (high radiation) applications. In the next section, we will introduce a more appropriate de-rating methodology. When exposed to gamma ray and

ultraviolet radiation, there are insulation aging degradations that are similar in affect to thermal radiation aging. The following section introduces alternate methods for dealing with the effect of ionizing radiation aging of electrical components.

5.2.4 Radiation De-rating of Winding Assemblies

As denoted above, radiation has similar effects electrical components that thermal radiation. Two methods based on similar principles were developed to handle the effects of ionizing radiation aging. The first method (ψ) accounts for both the amount of radiation absorbed and the rate at which the radiation is absorbed. The second (γ) incorporates the amount of radiation absorbed.

The first method is to calculate an effective temperature from the incident radiation and ambient temperature, with the components approximated as a black body. Thermal radiation is taken into account with T_a

$$T_{eff} = \sqrt[4]{T_a^4 + \psi \cdot \zeta} \dots\dots\dots(6)$$

Where,

- T_a = ambient or operating temperature (K) T
- ζ = incident space radiation / σ_B (space radiation is about 7 W/m²)
- σ_B = Stefan-Boltzmann constant 5.67E-08
- ψ = Harrington Psi Life Function (proposed), de-rates electrical and insulation components

$$\psi(\delta) = a \cdot \delta^b \dots\dots\dots(7)$$

- δ = Total ionizing dose of space radiation during the mission (rad)
- a = proposed constant, for Teflon insulation applications, $a = 1.989$
- b = proposed constant, for Teflon insulation applications, $b = 0.178$

The constant in Eq. 7 arises from the success of the chosen insulation system. The equation is used to add an additional margin of safety to the degradation of electrical components. Introducing ϵ the exposure ratio, where:

$$\epsilon = \frac{L_o}{L_M} \dots\dots\dots(8)$$

- L_o = Electrical Operating Time in Hours
- L_M = Mission Life in Hours

Finally, combing the results into the Brancato Method:

$$L_w = 100 * (\epsilon * 2^{\frac{T_R - T_{effo}}{10}} 10 + (1 - \epsilon) * 2^{\frac{T_R - T_{effi}}{10}}) \dots\dots\dots(9)$$

Where,

- L_w and T_R can be found following Eqn. 5

While this method is robust it requires a bit of calculation, instead an exponential decay function that provides a de-rating factor is far simpler. An exponential function was chosen based on information about the change in physical and electrical properties of materials exposed to radiation in Ref [7]. Which denotes a decrease in bulk resistivity, a critically property of insulation systems, is an exponential decay relationship to the increase in radiation dose. The de-rating factor can be calculated by using Eq. 10

$$\gamma = e^{-\delta/\rho} \dots\dots\dots(10)$$

Where,

- γ = Harrington Gamma Life Function (proposed), de-rates insulation life due to radiation dose
- δ = Total ionizing dose of space radiation during the mission (rads)
- ρ = lifetime constant, 4342944 for Teflon, results in .01 attenuation at the ‘threshold’ of radiation damage

Eq. 10 de-rates the averaged Brancato life as seen in Eq. 9, but instead T_{eff} is replaced by T_a and T_b . See Appendix D for the generalized form.

The first method (ψ) presented is robust and extremely conservative compared to method two (γ) up to 1.5E+07 rads, at which point it is less conservative. The second method is simple to calculate. Both methods were designed to derate electrical component life to ~1% at the threshold of radiation-induced degradation.

5.3 Reliability Analysis Toolkit:

Using the on-line tools of the Reliability Analysis Toolkit, Ref. [2], we are able to predict the reliability failure rate of the actuator. The methods used in this approach are derived from the NSWC Handbook of Reliability Prediction Procedures for Mechanical Equipment. The online tools, using the equations described in 5.3.2 applicable Weibull distribution functions for different components.

5.3.1 Reliability Equations:

Using the L_2 to MTBF Conversion, the equations represent a three-parameter Weibull distribution, with δ shown in the equations; however, for purposes of this tool this parameter (sometimes called "failure free life") is assumed to be zero, and eliminated from these equations. Note, the variables and symbols used in this section are consistent with those used in Ref. [2].

Reliability Function R(t):

$$R(t) = e^{-\left(\frac{t}{\eta}\right)^\beta} \dots\dots\dots(11)$$

Probability Density Function f(t):

$$f(t) = \frac{\beta(t)^{\beta-1}}{\eta^\beta} e^{-\left(\frac{t}{\eta}\right)^\beta} \dots\dots\dots(12)$$

h(t), instantaneous failure rate:

$$h(t) = \frac{f(t)}{R(t)} \dots\dots\dots(13)$$

The average failure rate is calculated using the Eq. 14:

$$\lambda_{Average} = \frac{1 - R(T)}{\int_0^T R(t) dt} \dots\dots\dots(14)$$

System Input Parameters:

1. Shape parameter (β): Weibull shape “Beta” factors. From Ref. [2]. Usages of Typical or High End factors are provided. A Beta factor of 1.2 is used for windings, 1.3 for Ball Bearings and 2.0 for Gearboxes. Higher complexity assemblies utilize higher β factors.

2. Characteristic Life (η) is the 63.2% failure point for a mechanical system. The Characteristic Life is determined by the L_2 life and the β factor. For instance, for a β of 1.3, as for a bearing, the Characteristic Life is about 20x the L_2 life. For a β factor of 2.0, the multiplication factor is about 7x the L_2 life. η can be calculated exactly by using Eq. 15.

$$\eta = L_n \left(-\ln \left(1 - \frac{n}{100} \right) \right)^{-1/\beta} \dots\dots(15)$$

3. Maintenance interval for item renewal (T): No maintenance of this product is required or assumed. A value of 100 years is used in the on-line tool equations.

5.3.2 MTBF with the Reliability Toolkit:

The on-line Reliability Toolkit was used to generate the MTBF, from the β function and the L_2 predicted life. The lifetime estimations of the bearings and gears are based on conservative de-ratings of fatigue and wear-out considerations, with margin, the failure rate of these estimations will be proportional to the required life over the predicted life. This method has been used in reliability analysis presented to, and accepted by many other programs and subject matter experts in the industry. The justification for this analysis methodology is the additional manufacturing precision, workmanship testing and quality inspections that take place for space

flight hardware to eliminate infant mortality and other workmanship issues that affect commercial hardware.

MTBF can be calculated using Eq. 16

$$MTBF = \frac{\frac{\eta}{\beta} \Gamma\left(\frac{1}{\beta}, \left(\frac{T}{\eta}\right)^\beta\right)}{1 - e^{-\left(\frac{T}{\eta}\right)^\beta}} \dots\dots\dots(16)$$

where,

- $\Gamma(a,x)$ is the incomplete gamma function

Equation 11 can be ‘altered’ by setting $MTBF = T$. By doing so, a ‘No Maintenance’ interval is achieved i.e. the part on average would fail at the maintenance interval. See Appendix C for the method and code.

5.3.3 Redundancy Factor for Windings:

Since there is selectable redundancy for the motor and resolver windings, we may use the Redundancy Factor described in the Reliability Toolkit. We use the Effective Failure Rate on One Standby Offline Unit with One Active On-line Unit Required for Success (without repair), See Eq. 20.

5.4 Reliability Number:

We essentially have two separate life analyses to consider. The mechanical loaded-life (t_M) for the application is analyzed for 318 hours of operation. The windings and the electrical insulation system are exposed to radiation the entire time in orbit, so regardless of powered-on time we assume the time on orbit degrades the insulation system. For a five-year orbital mission for a high-radiation, geosynchronous application, we have an electrical life requirement (t_E) of 44,000 hours of operation and $2E+04$ Rads Total Ionizing Dose (TID) was used for total radiation during mission life. Refer to Appendix A for a table of analysis parameters.

To calculate the Mechanical Reliability for the operational profile from the MTBF, Eq. 17 is used:

$$R_{IM} = e^{\left[\frac{-t_M}{MTBF_M}\right]} \dots\dots\dots(17)$$

Where,

- R_{IM} = Mechanical Reliability
- t_M = Time of loaded operation, in Hours (318)
- $MTBF_M$ = Mechanical Mean Time Between Failures (1.20E+05)

This results in an $R_{IM} = 0.99736$

Similarly, the Electrical Reliability is calculated by:

$$R_{IE} = e^{\left[\frac{-t_E}{MTBF_E}\right]} \dots\dots\dots(18)$$

Where,

- R_{IE} = Electrical Reliability
- t_E = Time of exposure, in Hours (44,000)
- $MTBF_E$ = Electrical Mean Time Between Failures using (8.95E+12) calculated using γ de-rated Averaged Brancato method.

Which results in an $R_{IE} = 0.99999$

And the overall reliability (R_O) is calculated by:

$$R_O = R_M \cdot R_E \dots\dots\dots(19)$$

Therefore, the overall reliability $R_O = 0.99736$ is achieved.

The MTBF for n redundant units, 1 spare with no repair interval:

$$MTBF_R = MTBF * n * (P + 1) \dots\dots\dots(20)$$

Where,

- n = number of active units
- P the probability that the spare will work

The probability of the spare working can be treated as that units’ reliability. Using these equations for both cases of the electrical components results in $R_O = .99736$ for the ψ factored electrical decay. See Appendices A and B for more information on reliability values. If we needed to increase the resultant overall reliability, we would address the two lowest MTBF components. In this case, the Mid-Speed Carrier Bearings that see the radial and thrust loads from the bevel mesh and the low-speed planetary module are the reliability-driving components. Possible methods to increase reliability in these components is to increase the gearbox and bearing size, or possibly change materials to increase the L_2 ratings.

6. QUALIFICATION AND LIFE TESTING

The environmental qualification and life endurance tests were required for the subject actuator. After vibration testing and thermal cycling, a life test was conducted. Of particular concern was the number of cycles required at the high-speed motor. As shown in Fig. 3, the

actuator is coupled to the simulated load inertia through a torque transducer and bearing support system. The position output of the resolver was processed and recorded to a data recorder to assure no missed steps or transients were observed in the testing. The life test was conducted at room temperature as well as the high and low temperature extremes. All qualification tests were successful, and the actuator is integrated into the payload, awaiting launch.

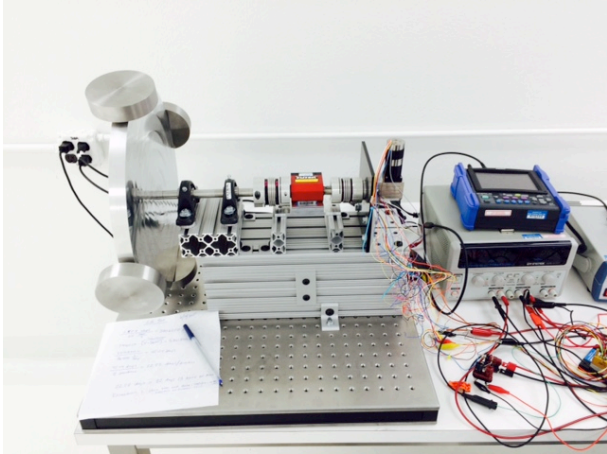


Figure 3

Life Test Set-Up with 0.5 kgm² Load Inertia

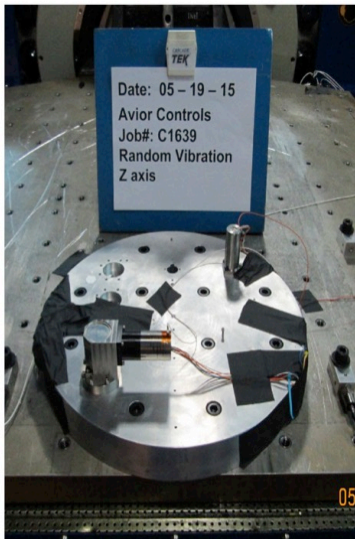


Figure 4

Random Vibration Qualification Test

7. CONCLUSION

Requirements for torque margin may have structural and endurance implications for transmitted torque through the mechanism that must be considered. Non-current limited applications may also have significant torque consequences at extreme conditions that are often not analysed. The paper also details a methodology for calculating reliability using approved

analysis techniques and methodologies. We also introduce two types of analysis for de-rating life estimates for motor insulation systems, due to insulation aging effects due to prolonged exposure to radiation.

The subject actuator has successfully completed all qualification testing and the instrument is fully integrated into the system, and is awaiting delivery.

8. REFERENCES

1. "Primer – Stepper Motor Nomenclature, Definition, Performance and Recommended Test Methods" Starin et al. Proceedings of the 42nd Aerospace Mechanisms Symposium, NASA Goddard Space Flight Center, May 14-16, 2014
2. <http://reliabilityanalyticstoolkit.appspot.com/>
3. "Estimation of Lifetime Expectations of Motors" Emanuel L Brancato, IEEE Electrical Insulation Magazine, May/June 1992-Vol. 8 No.3
4. "Analyze of Reliability of Gears" Dr. N. Ungureanu et al
5. "How to Determine the MTBF of Gearboxes" Dr. Gerhard G. Antony Neugart, Paper for 2007 AGMA FTM
6. NSWC Handbook of Reliability Prediction Procedures for Mechanical Equipment, May 2011.
7. "Nuclear and Space Radiation Effects on Materials", NASA, July 1970.

Appendix A – MTBF and Reliability Table - Mechanical						
A	B	C	D	E	F	G
Subsystem Module or Component	Beta (Weibull Shape Parameter)	L ₂ Life (Hours)	Characteristic Life (Hours)	MTBF (Hours)	Average Failure Rate (Hours)	Reliability
N/A	β	See 5.2	η	Ref. [2]	λ	Eq. 5
Motor Rotor Bearing	1.3	4.50E+05	9.05E+06	9.57E+06	1.05E-07	0.99997
High Speed Planetary Module	2	1.00E+06	7.04E+06	7.79E+06	1.28E-07	0.99996
High Speed Planet Gear Bearings	1.3	1.00E+05	2.01E+06	2.13E+06	4.70E-07	0.99985
High Speed Carrier Bearings	1.3	5.27E+05	1.06E+07	1.12E+07	8.93E-08	0.99997
Mid Speed Planetary Module	2	1.00E+06	7.07E+06	7.79E+06	1.28E-07	0.99996
Mid Speed Planet Gear Bearings	1.3	8.75E+05	1.76E+07	1.86E+07	5.38E-08	0.99998
Mid Speed Carrier Bearing	1.3	2.94E+04	5.91E+06	6.25E+05	1.60E-06	0.99949
Right Angle Bevel Gears	2	1.00E+06	7.04E+06	7.79E+06	1.28E-07	0.99996
Right Angle Carrier Bearing	1.3	3.17E+07	6.38E+08	6.74E+08	1.48E-09	0.99999
Right Angle Support Bearing	1.3	5.40E+06	1.09E+08	1.15E+08	8.71E-09	0.99999
Low Speed Planetary Gearing	2	2.50E+04	1.76E+05	1.95E+05	5.14E-06	0.99837
Low Speed Planet Gear Bearings	1.3	1.03E+05	2.07E+06	2.19E+06	4.57E-07	0.99985
Low Speed Carrier Bearings	1.3	2.34E+06	4.71E+07	4.97E+07	2.01E-08	0.99999
			Total	1.20E+05	8.33E-06	0.99736

Appendix B – MTBF and Reliability Table - Electrical							
A	B	C	D	E	G	H	I
Subsystem Module or Component	Beta (Weibull Shape Parameter)	L ₂ Life (Hours)	Characteristic Life (Hours)	MTBF (Hours)	Reliability	R _o	Redundant R _o
.1 Avior standard* at te=44000	1.2	2048000	5.29E+07	2.76E+07	0.99841	0.99577	0.99656
.1 Averaged Brancato at te=44000	1.2	2.60E+09	2.69E+10	1.40E+10	0.99999	0.99736	0.99736
ψ at te=44000	1.2	1.47E+11	3.80E+12	1.98E+12	0.99999	0.99736	0.99736
γ Averaged Brancato at te=44000	1.2	6.63E+11	1.13E+13	8.95E+12	0.99999	0.99736	0.99736

Calculations of reliability and MTBF include both the On Axis Resolver and Motor Windings

*The Avior “old” standard is the Brancato L₂ life at 120°C multiplied by .1
The averaged Brancato method can be found in Eq. 9 and in Appendix D
A value of -30°C was used for the cold or non-powered state of electrical components
A value of +120°C was used for the hotspot temperature of electrical components

Appendix C

The following code was written in Anaconda Spyder to calculate η , MTBF, λ , and no maintenance interval (T)
denotes code comment

```

from scipy.special import gammainc, gamma
from math import *
from math import exp as e

B =  $\beta$  #beta shape parameter
L2 =  $L_2$  #L2 life
n =  $L2 * (-\log(.98))^{**}(-1/B)$  # $\eta$ , characteristic life for L2
le =  $5 * 365 * 24$  #mission life for electrical components (Hours)
lm = 318 #usage life requirement for
T = 50 #initial guess in hours of no maintenance interval
R =  $e^{-(T/n)**B}$  #reliability function
MTBF =  $(n/B * \text{gamma}(1/B) * \text{gammainc}(1/B, (T/n)**B)) / (1-R)$  #see ( $\diamond$ )
err =  $\text{abs}(T - \text{MTBF}) / \text{MTBF} * 100$  #percent error
ep = .1 #final percent error

while err > ep: # subroutine solves T for conditions stated after Eq. 11 by simply convergence
    T = MTBF # Iterated map on T
    R =  $e^{-(T/n)**B}$ 
    MTBF =  $(n/B * \text{gamma}(1/B) * \text{gammainc}(1/B, (T/n)**B)) / (1-R)$ 
    err =  $\text{abs}(T - \text{MTBF}) / \text{MTBF} * 100$ 

print(n, MTBF, 1/MTBF, e-lm/MTBF) #reports results  $\eta$ , MTBF,  $\lambda$ , and reliability  $R_n$ 
print(T/365/24, "Years") #no maintenance life in years

```

\diamond scipy gammainc is normalized, so it has to be multiplied by gamma

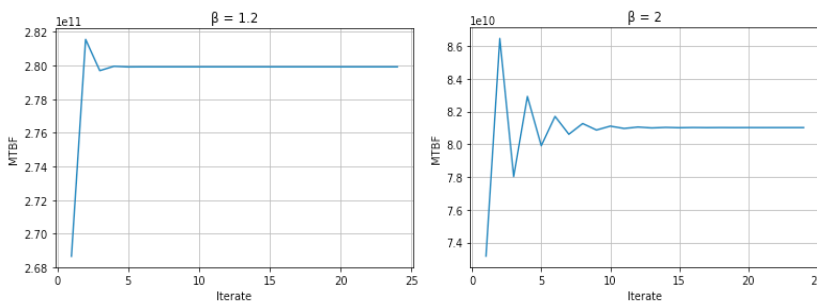
$\xi(T)$ is a non-linear iterated map in 1 dimension. T is the iterated value and β the driving constant.

$$\xi(T_n) = \frac{1}{1 - e^{-(\frac{T}{\eta})^\beta}} \frac{\eta}{\beta} \Gamma\left(\frac{1}{\beta}, \left(\frac{T}{\eta}\right)^\beta\right)$$

$$T_{n+1} = \xi(T_n)$$

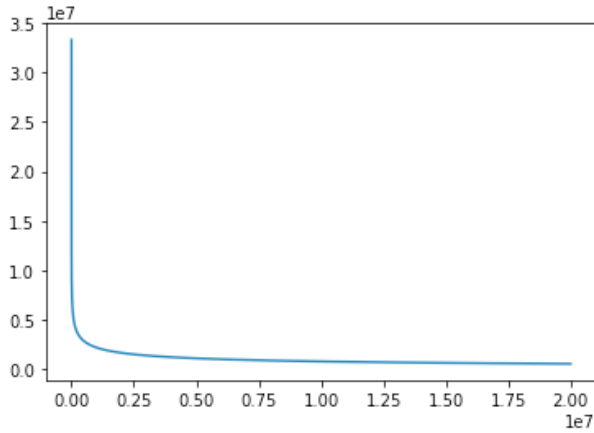
Additionally, fixed point iterations of the map exist between,

$0 < \beta$ and $\sim \beta \leq 2.883$ at $\beta > 2.883$ the map undergoes period doubling and the limit cycle iterations of the map become a period 2 orbit. Solutions are around the same order as η . Choosing η as an initial guess provides faster convergence by about 1 iteration. Below are several graph of iteration for two values of β . For $\beta=1.2$ the solution converges within .1% in 3 iterations and for $\beta=2$ the solution converges to within .1% in 12 iterations.

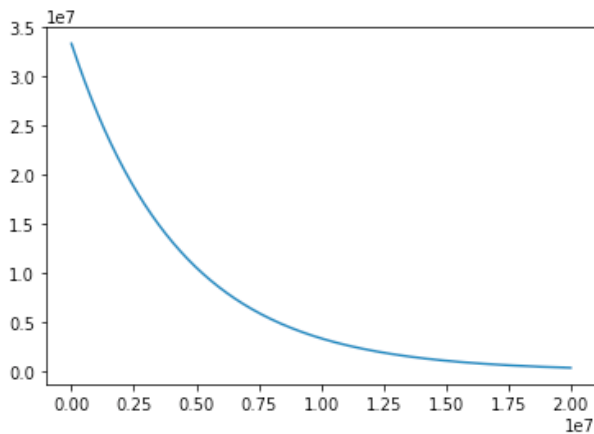


For any additional question about the convergence of ξ , please contact the authors.

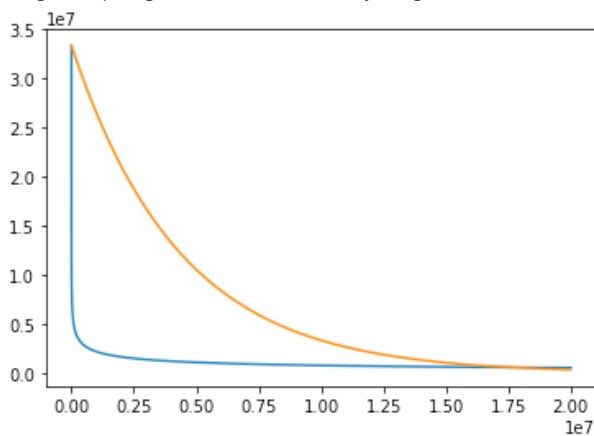
Appendix D



Graph of ψ implemented Life decay, Eq. 9



Graph of γ implemented Life decay, Eq. 10



Both Eq. 9 and 10 together

Averaged Broncato Equation (Generalized Equation 9),

$$L_w = 100 * \sum_i \varepsilon_i * 2^{\frac{T_R - T_i}{10}}$$

Where,

$$\sum_i \varepsilon_i = 1$$

- T_i = some temperature
- ε_i = percent exposure to T_i